

# Optimal Transfer Mechanism for Municipal Soft Budget Constraints in Newfoundland

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## Abstract

Newfoundland and Labrador’s municipalities face severe soft budget pressures due to narrow tax bases, high fixed service costs, and volatile resource revenues. We develop a Stackelberg style mechanism design model in which the province commits at  $t = 0$  to an *ex ante* grant schedule and an *ex post* bailout rule. Municipalities privately observe their fiscal need type, choose effort, investment, and debt, and may receive bailouts when deficits exceed a statutory threshold. Under convexity and single crossing, the problem reduces to one dimensional screening and *admits* a tractable transfer mechanism with quadratic bailout costs and a statutory cap. The optimal *ex ante* rule is *threshold-cap*; under discretionary rescue at  $t = 2$ , it becomes *threshold-linear-cap*. A knife-edge inequality yields a self-consistent no bailout regime, and an explicit discount factor threshold renders hard budgets dynamically credible. We emphasize a class of monotone *threshold* signal rules; under this class, grant crowd out is null almost everywhere, which justifies the constant grant weight used in closed form expressions. The closed form characterization provides a policy template that maps to Newfoundland’s institutions and clarifies the micro-data required for future calibration.

**JEL:** H71; H77; D82.

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# 1 Introduction

Local public finances in Newfoundland and Labrador (NL) have long faced a three-way squeeze: a limited own-source tax base, large fixed costs of delivering services to sparsely populated areas, and the province’s chronic macro-fiscal stress.<sup>1</sup> Few Canadian provinces illustrate the political economy of *soft budget constraints* (SBCs) so vividly as NL.

SBC, described by Kornai (1986), arise once an upper-tier government develops a bailout record and lower-tier entities rationally expand spending or borrowing in expectation of future relief. On the revenue side, sparsely populated outports yield a narrow own-source tax base; on the cost side, geography and winter logistics impose one of the highest per-capita service bills in the country. When commodity downturns hit—most recently in 2015–2016—the province repeatedly “stepped in”: it assumed municipal debts *in toto*, stretched repayment schedules, and negotiated ad hoc funding packages with Ottawa.<sup>2</sup> By normalizing such rescues NL has created precisely the expectation of future relief that Kornai (1986) warned about.

Although a rich literature analyzes SBCs in transition economies and US/EU federal systems, two limitations stand out:

- (i) **Timing.** Most formal models cast upper and lower tiers as simultaneous Nash players; real-world transfers are decided *sequentially*.
- (ii) **Granularity.** Empirical work on Canada concentrates on provincial–federal equalization; the municipal layer—where soft budgets can first bite—has received scant theoretical attention (Bird, 2012; Sancton, 2014; Boothe, 2015).

In this paper, we build a *Stackelberg-style screening model* in which the province first commits to a two-part menu  $(T(\hat{\theta}), b(\hat{\theta}))$ ; a municipality then privately observes its *fiscal need* type  $\theta$  and reports  $\hat{\theta}$ ; finally the province may—*ex post*—augment the transfer with a bailout schedule  $\beta(\cdot)$  that maps the noisy gap signal  $\hat{G}$  to a payment. We adopt the **implementable payout convention**

$$\text{Realized payout } p(\hat{G}, \hat{\theta}) = \mathbf{1}\{\hat{G} > 0\} \cdot \min\{\beta(\hat{G}), b(\hat{\theta}), \hat{G}\},$$

so that the *signal-based rule*  $\beta$  is hard-capped by the *type-based cap*  $b$  and by the observed signal. This aligns the model with administrative practice, avoids paying above the observed gap, and acknowledges that mis-payment risk stems only from signal noise.

The model is theoretical, yet four closed-form results offer a tractable benchmark for practitioners and future empirical work in NL:

- (a) **Dimensionality reduction.** The three-stage game collapses to one-dimensional adverse selection with two scalar instruments  $(T, b)$ ; see Proposition 4.8.

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<sup>1</sup>NL’s per-capita debt-service burden has ranked first among Canadian provinces since at least 2020; see Government of Newfoundland and Labrador (2020).

<sup>2</sup>See the 25 May 2016 press release, Government of Newfoundland and Labrador (2016).

- (b) **Optimal transfer rule and knife-edge.** With linear-quadratic provincial costs and a statutory cap, the IC-IR-LL optimum is *threshold-cap*; under a discretionary  $t=2$  rescue (no commitment), the realized rule is *threshold-linear-cap* (Appendix C). A self-consistent no-bailout regime obtains whenever

$$\boxed{\alpha\omega_T \geq \gamma\omega_b \cdot \sup_{\theta} h(\theta)}, \quad h(\theta) = \frac{f(\theta)}{F(\theta)},$$

see Proposition 4.13 and Barlow and Proschan (1975); Shaked and Shanthikumar (2007).

- (c) **Dynamic credibility.** Embedding the game in an infinite horizon yields the discount-factor threshold

$$\rho^* = \frac{\omega_T - \omega_b}{\kappa + \gamma + \omega_T - \omega_b},$$

which renders a hard budget self-enforcing (Appendix D; Abreu, 1988; Fudenberg and Tirole, 1991).

- (d) **Policy template for NL.** The resulting two-parameter grant formula  $(\theta^{\min}, \theta^{\dagger})$  maps to NL’s Municipal Operating Grant and identifies the micro-data needed to calibrate  $(\gamma, \kappa, \lambda_T, \alpha, \omega_b)$ ; see Proposition 4.12.

*Roadmap.* Section 4 develops the Stackelberg model and derives the reduced form. Section 4.7 solves for the optimal transfer schedule and proves its second-best efficiency, introducing the thresholds  $\theta^{\min}$  and  $\theta^{\dagger}$  (defined in (4.8)). Section 5 draws policy lessons and comparative statics for NL. Section 6 concludes and outlines empirical extensions.

## 2 Literature Review

The paper intersects three strands of work: (i) soft budget constraints (SBC) in multi-tier public finance, (ii) mechanism design with Stackelberg leadership, and (iii) Canadian municipal-finance empirics.

### 2.1 Soft Budget Constraints

The SBC idea begins with Kornai (1986). Early formalizations (Kornai et al., 2003) show that ex post efficient bailouts undermine ex ante effort and borrowing; see also Dewatripont and Maskin (1995) for a dynamic commitment model. Applications to intergovernmental finance include Weingast (1995) on U.S. states, Bordignon et al. (2001) on European stability pacts, and the survey by Goodspeed (2016). Recent papers ask *when* an upper tier can credibly refuse rescues: Amador et al. (2021) derive fiscal limits under limited commitment, Pavan and Segal (2023) study repeated screening, and Acemoglu and Jackson (2024) analyze relational contracts with hidden actions. Yet these models stop short of a closed-form, policy-ready transfer rule. We fill that gap by producing an implementable

*threshold-cap* schedule under commitment (and *threshold-linear-cap* under  $t=2$  discretion) and a single discount-factor test for self-enforcing hard budgets.

## 2.2 Mechanism Design and Stackelberg Leadership

Incentive-compatible grant design dates back to Bordignon et al. (2003), who use a single matching grant under simultaneous moves. Toma (2013) introduces leader–follower timing but assumes full information. Chen and Silverman (2019) obtain threshold payments in a one-shot health-care model with asymmetric costs, yet ignore ex post instruments and dynamic credibility. Our contribution is twofold: (i) a *two-instrument* Stackelberg screen that nests bailout and no bailout regimes in the marginal-cost inequality  $\alpha\omega_T \gtrless \gamma\omega_b$  (with the exact knife-edge refined by the hazard bound (4.13)); (ii) a closed-form critical discount factor that pins down dynamic self-enforcement.

## 2.3 Canadian Municipal Finance

Empirical work on Canadian municipalities examines fiscal capacity and service costs (Bird, 2012; Sancton, 2014) or tax-base sharing (Dahlby and Ferde, 2021), and analyzes borrowing limits (Found and Tompson, 2020). Evidence on municipal-level SBC is scarce; most studies focus on federal–provincial equalization (Boothe, 2015). A notable exception is Bracco and Doyle (2024), who exploit BC’s 2004 debt-limit reform, and the cross-country benchmark in Rodden (2006). None, however, links provincial bailout policy to a mechanism-design benchmark. Our theory provides that benchmark and outlines the data needed for future identification once municipal micro-panels for NL—or other provinces—become available.

# 3 Provincial–Local Fiscal Relations in Newfoundland and Labrador

Although the model is self-contained, its parameters map neatly onto four institutional frictions that characterize NL. Documenting those frictions clarifies both the choice of primitives and the comparative-statics exercises that follow.

## 3.1 A Heterogeneous Local Landscape

In NL, more than 260 local units fall into two legal categories. Incorporated municipalities possess broad tax powers—chiefly the property tax—whereas unincorporated Local Service Districts (LSDs) finance specific services through earmarked levies. LSDs are typically small, sparsely populated, and administratively thin; the province therefore faces persistent political pressure to guarantee minimum service levels even when the local tax base is weak. We model this dispersion of own-source capacity as private information: each jurisdiction’s *fiscal*

*need type*  $\theta$  indexes the severity of its funding gap (higher = weaker capacity / larger need).

### 3.2 Transfer Instruments and Model Notation

Three provincial channels matter and correspond one-for-one to our model variables:

**Municipal Operating Grant (MOG).** An *unconditional* operating transfer that forms the baseline grant  $T(\theta)$ .

**Canada Community–Building Fund (CCBF).** Formula-based or cost-shared capital money. Because these flows are largely exogenous to short-run fiscal gaps we treat them as a constant background term  $g$  and abstract from them in the formal screening problem.

**Special Assistance.** Ad-hoc subsidies or debt relief for distressed communities; this is the discretionary bailout instrument  $b(\theta)$ .

### 3.3 Borrowing Oversight and Commitment Frictions

Long-term municipal borrowing requires ministerial approval and often carries an explicit provincial guarantee. Once a community approaches default the province internalizes the externality of municipal bankruptcy and almost always chooses rescue over liquidation. Anticipating that bias, low-capacity jurisdictions rationally relax ex ante effort—exactly the soft budget channel formalized in our Stackelberg mechanism.

### 3.4 Stylized Fiscal Facts and Parameter Guidance

- (a) **High provincial debt service.** NL’s per-capita debt-service burden tops the Canadian league table, implying a high marginal opportunity cost  $\gamma$  of each grant dollar.
- (b) **Transfer-dependent small jurisdictions.** For many LSDs, unconditional grants finance over half of current spending, whereas bailouts are politically constrained and arrive late. Thus  $\omega_T > \omega_b$ .
- (c) **Administrative capacity gaps.** Uneven record-keeping and audit lags create the information asymmetry that justifies the single-crossing (or virtual monotonicity) used in the model.
- (d) **Prohibitively high political cost of very large rescues.** Public debate in NL treats bailouts above a “headline” threshold as politically costly, consistent with the convex cost term  $\frac{\kappa}{2}b^2$  in the provincial objective.

*From institutions to the model.* Sections 4–4.7 fold the institutional environment into a two-instrument screen  $(T, b)$ : an ex ante operating grant and an ex post bailout. All primitives that do not affect screening incentives directly (e.g. CCBF capital transfers  $g$ ) are absorbed into fixed terms.

## 4 Model

We study the interaction between a single provincial government  $P$  and a continuum of local jurisdictions  $i \in \mathcal{I} \subset [0, 1]$  representing municipalities or Local Service Districts (LSDs).

**Type convention.** Throughout, we define the private type  $\theta \in [\underline{\theta}, \bar{\theta}]$  as a *fiscal-need index*: a higher  $\theta$  corresponds to a weaker local tax base / higher per-unit service cost, hence a larger underlying funding gap. This convention is used consistently in the theory and background sections.

### 4.1 Technologies and Preferences

**Basic services** Each local jurisdiction  $i$  delivers a bundle of essential public services  $q = (\text{water, sewer, roads, } \dots)$ . The monetary cost is modeled by  $C(q, \theta)$ , where  $\theta$  denotes the jurisdiction's fiscal need.

*Higher  $\theta$  (narrow tax base, difficult geography)  $\Rightarrow$  higher marginal cost.*

**Heterogeneous fiscal need** Fiscal need is heterogeneous across municipalities. We treat  $\theta$  as a draw from a continuous distribution  $f(\theta)$  with support  $[\underline{\theta}, \bar{\theta}]$ , so both high- and low-need jurisdictions are present in the province.

**Local effort** Jurisdictional effort  $e \geq 0$  — tax enforcement, fee collection, grant writing — generates own-source revenue  $R(e, \theta)$  with  $R'_e > 0$  and  $R''_{ee} < 0$ .

**Disutility of effort** Effort imposes a linear utility cost

$$\text{Disutility} = -\phi e, \quad \phi > 0,$$

on residents or officials. The linear form keeps derivations tractable while capturing the political and administrative cost of higher taxation.

**Service and investment utility** Delivering services and undertaking investment also generate direct benefits:  $B(q)$  captures household utility from the service bundle  $q$ , while  $\Gamma(I)$  is the longer-run payoff from capital outlay  $I$ . Both are twice differentiable with diminishing marginal returns and enter the objective only through boundedness assumptions.

**Transfers** The Province can effect intergovernmental transfers in three distinct ways:

- (i) **Unconditional grant**  $\tau$ , set *ex ante* before local effort choices;
- (ii) **Matching transfer** at provincial share  $s$  on capital outlays  $I$ ;

- (iii) **Ex-post bailout**  $b \geq 0$ , extended only if a realized gap remains after fiscal shocks  $\varepsilon$  are realized.

Before local choices, the Province commits to three *transfer instruments*. Table 1 summarizes timing and incentives.

Given these transfers, the **one-period cash-flow constraint** at  $t = 1$  (before any payout) is

$$G = [C(q, \theta) + (1 - s)I + rD] - [R(e, \theta) + \tau + g + sI + D] + \varepsilon \quad (4.1)$$

3

If  $G > 0$  a funding gap exists. Province observes a noisy signal  $\hat{G} = G + \eta$  and pays the *realized payout*

$$p(\hat{G}, \hat{\theta}) = \mathbf{1}\{\hat{G} > 0\} \cdot \min\{\beta(\hat{G}), b(\hat{\theta}), \hat{G}\} \in [0, \hat{G}],$$

at  $t = 2$  under commitment to  $(\beta, b)$ .

**Softness metric** We define the *softness index* as the ex-ante rescue probability

$$\pi = \mathbb{P}[p(\hat{G}, \hat{\theta}) \geq G] \in [0, 1],$$

namely the *probability that a realized funding gap will be fully covered*:  $\pi = 0$  is a hard budget constraint;  $\pi = 1$  is full insurance.

**Aggregate softness.** Throughout Sections 4.2, 4.5, and 4.6 we treat  $\pi$  as the *aggregate* ex ante rescue probability (integrated over types). It is a descriptive index; in the baseline with threshold  $\beta$  it does not affect the grant weight derived below.

## 4.2 Local effort $e$ : incentives and marginal condition

Effort  $e \geq 0$  (tax enforcement, fee collection, administrative intensity) generates own-source revenue  $R(e, \theta)$  with  $R'_e > 0$  and  $R''_{ee} < 0$ . Effort is personally/politically costly as  $-\phi e$ .

**Assumption 4.1** (Primitives). *Types  $\theta$  lie in  $[\underline{\theta}, \bar{\theta}]$  with density  $f > 0$ . The local cost and revenue functions satisfy, for all  $\theta$ ,*

$$C'_q(q, \theta) > 0, \quad C''_{qq}(q, \theta) \geq 0, \quad R'_e(e, \theta) > 0, \quad R''_{ee}(e, \theta) < 0.$$

**Assumption 4.2** (Observation & rule regularity). *The signal rule  $\beta$  is non-decreasing and a.e. differentiable with slope in  $[0, 1]$ ; the audit noise  $\eta$  has a continuous density  $f_\eta$  with bounded tails; and  $(e, \theta) \mapsto R'_e(e, \theta)$  is continuous. These ensure interchange of expectation and differentiation and validate the marginal-probability formulas below.*

<sup>3</sup>Notation reminder: in the reduced-form mechanism we write  $T(\theta)$  for the unconditional operating grant that corresponds to the empirical instrument  $\tau$ ; and  $g$  captures largely exogenous capital transfers (e.g. CCBF/Gas-Tax) that are treated as constants in screening.

**Assumption 4.3** (Signal MLRP). *For  $\theta_2 > \theta_1$ , the family of signals  $\{\hat{G} \mid \theta\}$  satisfies the monotone-likelihood-ratio property (MLRP), so that for any nondecreasing  $\varphi$ ,  $\mathbb{E}[\varphi(\hat{G}) \mid \theta_2] \geq \mathbb{E}[\varphi(\hat{G}) \mid \theta_1]$ . Since  $\beta$  is nondecreasing, the cap-slope objects defined below are nondecreasing in  $\theta$ .*

**Assumption 4.4** (Restriction to threshold signal rules). *We restrict attention to signal-based payout rules  $\beta$  that are nondecreasing and piecewise constant in  $\hat{G}$  (threshold rules). This class is consistent with administrative practice and eliminates effort-report interactions almost everywhere.*

**Observation frictions and default.** Default occurs iff  $p(\hat{G}, \hat{\theta}) < G$ . Because of information frictions, municipalities rationally expect a positive rescue probability  $\pi > 0$ . Anticipating the chance of a bailout, they optimally reduce tax effort  $e$  and rely more on debt  $D$ .<sup>4</sup>

**Assumption 4.5** (Effort independence via threshold  $\beta$ ). *Under Assumptions 4.1–4.4, the first-order condition for effort on the cap-slack branch contains no term depending on the report  $\theta$ ;  $e^*(\theta)$  is report-independent up to boundary-density terms on the cap-binding set.*

*Technical detail.* See Lemma H.1 in Appendix H.

### 4.3 Annual timeline: three decision stages

We normalize one fiscal year to  $t \in \{0, 1, 2\}$ ; the sequence repeats every year.

**Stage  $t = 0$  (policy commitment).** The province announces a vector  $\Pi = (\tau, s, \bar{D}, \beta, b)$  where  $\beta : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is the signal-based payout rule implemented at  $t = 2$  and  $b(\cdot)$  is a type-based cap.

**Stage  $t = 1$  (local choices and gap realization).** After observing its private type  $\theta \sim f$ , the municipality selects effort  $e \geq 0$ , capital  $I \geq 0$ , and debt  $0 \leq D \leq \bar{D}$ . A mean-zero fiscal shock  $\varepsilon$  is then realized and the pre-payout gap is  $G$  from (4.1).

**Stage  $t = 2$  (signal, payout, default test).** The Province observes  $\hat{G} = G + \eta$  and pays  $p(\hat{G}, \hat{\theta}) = \mathbf{1}\{\hat{G} > 0\} \min\{\beta(\hat{G}), b(\hat{\theta}), \hat{G}\}$ . Default occurs iff  $p(\hat{G}, \hat{\theta}) < G$ .

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<sup>4</sup>For axiomatic treatments of decision under noisy or imperfect perception, see Pivato and Vergopoulos (2020), which provides a clean way to model observation constraints consistent with our audit-noise setup.



#### 4.4 Local optimization at $t = 1$

Given policy  $\Pi$ , the municipality solves

$$\begin{aligned} \max_{e, q, I, D} \quad & \mathbb{E}_{\varepsilon, \eta} \left[ B(q) + \Gamma(I) + R(e, \theta) - \phi e - \varphi \mathbf{1}\{p(\hat{G}, \hat{\theta}) < G\} + \omega_b p(\hat{G}, \hat{\theta}) \right] \\ \text{s.t.} \quad & G = C(q, \theta) + (1 - s)I + rD - [R(e, \theta) + \tau + g + sI + D] + \varepsilon, \\ & 0 \leq D \leq \bar{D}, \quad e, q, I \geq 0. \end{aligned} \quad (4.2)$$

Here  $\omega_b > 0$  is treated as a constant marginal-utility weight that local decision-makers attach to each dollar of bailout; see Remark 4.5.

**Effort incentives via the default and marginal-rescue channels.** We report the marginal default formula for the *signal branch* (cap slack), which is the relevant case almost everywhere under threshold/linear  $\beta$ ; the cap only binds in an upper region where the marginal-rescue channel shuts down.

*Technical detail.* See Lemma H.2 in Appendix H.

Since  $\partial \mathbb{E}[p(\hat{G}, \hat{\theta})]/\partial e = -R'_e(e, \theta) \mathbb{E}[\beta'(\hat{G}) \mathbf{1}\{\beta(\hat{G}) < b(\hat{\theta})\}]$ , the interior FOC on the signal branch is

$$R'_e(e^*(\theta); \theta) \left\{ 1 + \varphi \mathbb{E} \left[ f_\eta(\hat{G} - \beta(\hat{G})) (1 - \beta'(\hat{G})) \right] - \omega_b \mathbb{E}[\beta'(\hat{G}) \mathbf{1}\{\beta(\hat{G}) < b(\hat{\theta})\}] \right\} = \phi. \quad (4.3)$$

For threshold (piecewise constant)  $\beta$ ,  $\beta'(\hat{G}) = 0$  a.e., hence the last term vanishes.

**Comparative statics.** From (4.3),  $\partial e^*/\partial \varphi > 0$  and  $\partial e^*/\partial f_\eta > 0$ . The sign of  $\partial e^*/\partial \omega_b$  is  $\leq 0$  through the  $-\omega_b \mathbb{E}[\beta'(\hat{G}) \mathbf{1}\{\beta < b\}]$  term; under threshold rules,  $\partial e^*/\partial \omega_b = 0$ .

#### 4.5 Reduction to One-Dimensional Screening

The three-stage environment features effort  $e$ , capital  $I$ , debt  $D$ , service  $q$ , and shocks  $(\varepsilon, \eta)$ . We now show that—under the convexity/monotonicity conditions already stated—these objects can be optimized out, leaving a one-dimensional adverse-selection problem with quasi-linear utility in  $(T, b)$ , where  $b$  is a *cap parameter*.

**Assumption 4.6** (Rare cap binding). *There exists  $\varepsilon < 1$  such that  $\mathbb{P}_\theta[\beta(\hat{G}) \geq b(\theta)] \leq \varepsilon$  for every feasible cap profile  $b(\cdot)$  in the optimal mechanism. This ensures the cap binds only on an upper tail with measure  $\leq \varepsilon$  and allows us to treat  $\tilde{b}(\hat{\theta}; \theta)$  and  $e^*(\theta)$  as  $O(\varepsilon)$ -close to the cap-slack expressions used in (4.4).*

*Clarification.* Assumption 4.6 refers to the probability (over the audit noise  $\eta$ ) that a given type's realization lies on the cap-binding branch; it does not exclude the existence of a type region  $[\theta^\dagger, \bar{\theta}]$  where the cap  $b(\theta) = \bar{b}$  binds ex ante.

**Step 1. Local optimization.** Fix  $(\tau, s, \bar{D}, \beta)$  and a true type  $\theta$ . Minimizing the pre-bailout resource block delivers  $(q^*, I^*, D^*)$  and reduced cost  $C_0(\theta)$ , independent of the report.

**Step 2. Effort choice.** Effort  $e^*(\theta)$  is pinned down by (4.3) (cap slack almost everywhere).

**Step 3. Quasi-linear reduced form with payout cap.** Define the *expected payout under cap*, conditional on type,

$$\tilde{b}(\hat{\theta}; \theta) = \mathbb{E}[\min\{\beta(\hat{G}), b(\hat{\theta})\} \mid \theta].$$

Then the interim utility from reporting  $\hat{\theta}$  can be written

$$U_L(\hat{\theta}, \theta) = \lambda_T(\theta) T(\hat{\theta}) + \omega_b \tilde{b}(\hat{\theta}; \theta) + K(\theta), \quad (4.4)$$

with a grant weight

$$\lambda_T(\theta) = \omega_T - \omega_b \mathbb{E}\left[\frac{\partial p}{\partial T} \mid \theta\right].$$

Under Assumption 4.4 and continuous noise,  $\beta'(\hat{G}) = 0$  almost everywhere, so  $\lambda_T(\theta) = \omega_T$ .

**Lemma 4.7** (Grant crowd-out factor). *With  $\hat{G} = G + \eta$  and  $G$  decreasing in  $T$  one-for-one, the marginal effect of  $T$  on the realized payout is*

$$\frac{\partial}{\partial T} \mathbb{E}[p(\hat{G}, \hat{\theta}) \mid \theta] = -\mathbb{E}\left[\beta'(\hat{G}) \mathbf{1}\{\beta(\hat{G}) < b(\hat{\theta})\} \mid \theta\right] + \text{boundary terms}.$$

*If  $\beta$  is threshold (piecewise constant),  $\beta'(\hat{G}) = 0$  a.e. and the boundary terms vanish under continuous noise, hence  $\lambda_T(\theta) = \omega_T$ . If  $\beta$  has an interior linear branch with slope  $m \in (0, 1)$  on the cap-slack set, then  $\lambda_T(\theta) = \omega_T - \omega_b m \cdot \mathbb{P}_\theta[\text{cap slack and } \hat{G} \text{ in linear range}]$ .*

**Proposition 4.8** (Reduction). *Under Assumptions 4.1, 4.2, 4.3 and 4.4, the original moral-hazard problem is equivalent to a direct mechanism in which municipal interim utility is quasi-linear in the cap parameter  $b$  through  $\tilde{b}(\hat{\theta}; \theta) = \mathbb{E}[\min\{\beta(\hat{G}), b(\hat{\theta})\} \mid \theta]$  as in (4.4).*

*Remark* (Constant marginal utility of bailouts). Lemma 4.7 implies that, under threshold  $\beta$ , the grant weight equals  $\omega_T$ . If instead a discretionary linear segment applies (Appendix C), the weight falls below  $\omega_T$  proportionally to the slope and the probability of being on the linear branch.

*Remark* (Effect of default-loss term and approximation accuracy). Because the default indicator only flips on the cap-binding set, which has probability at most  $\varepsilon$  by Assumption 4.6, the marginal effect of a report on the  $-\varphi \mathbf{1}\{p < G\}$  term is  $O(\varepsilon)$  and can be absorbed into  $K(\theta)$ . On the cap-slack region, the common tail probability cancels in the FOC just as in Lemma 4.11. This formalises the reduction in (4.4) up to an  $O(\varepsilon)$  error.

## 4.6 Single-Period Mechanism Design

We henceforth work with the reduced form (4.4), and with provincial cost taken in *expectation* over the realized payout.

**Provincial cost.** For a cap profile  $b(\theta)$  the expected per-type cost is

$$\mathbb{E} \left[ \alpha \min\{\beta(\hat{G}), b(\theta)\} + \frac{\kappa}{2} \min\{\beta(\hat{G}), b(\theta)\}^2 \mid \theta \right].$$

The province minimizes the population expectation of this cost plus  $\gamma T(\theta)$ , subject to IC, IR and limited liability.

**Envelope and weights.** With  $V(\theta) = U_L(\theta, \theta)$  and  $V(\underline{\theta}) = \underline{U}$ ,

$$V'(\theta) = \lambda'_T(\theta) T(\theta) + \omega_b \partial_\theta \tilde{b}(\theta; \theta) + K'(\theta). \quad (4.5)$$

*Remark (Baseline).* In the baseline with threshold  $\beta$  and constant  $\omega_b$ , we have  $\lambda_T(\theta) \equiv \omega_T$ , so (4.5) reduces to  $V'(\theta) = \omega_b \partial_\theta \tilde{b}(\theta; \theta) + K'(\theta)$ .

**Assumption 4.9** (Single crossing in the allocation index). *Define the allocation index for report  $\hat{\theta}$  at true type  $\theta$  by*

$$x(\hat{\theta}; \theta) = \lambda_T(\theta) T(\hat{\theta}) + \omega_b(\theta) \tilde{b}(\hat{\theta}; \theta),$$

*so that interim utility is  $U_L(\hat{\theta}, \theta) = x(\hat{\theta}; \theta) + K(\theta)$  with  $K$  absolutely continuous. Assume the single-crossing condition in  $x$ :*

$$\frac{\partial^2 U_L}{\partial \theta \partial x}(\hat{\theta}, \theta) \geq 0 \quad \text{for all } (\hat{\theta}, \theta).$$

*Under Assumption 4.3, this holds because  $\partial_\theta \tilde{b}(\hat{\theta}; \theta) \geq 0$  for nondecreasing  $\beta$ .*

**Problem 4.10** (Leader's program — reduced form).

$$\min_{T(\cdot), b(\cdot)} \mathbb{E}_\theta \left[ \mathbb{E} \left[ \alpha \min\{\beta(\hat{G}), b(\theta)\} + \frac{\kappa}{2} \min\{\beta(\hat{G}), b(\theta)\}^2 \mid \theta \right] + \gamma T(\theta) \right]$$

$$s.t. \quad \begin{cases} (IC) & V'(\theta) = \lambda'_T(\theta) T(\theta) + \omega_b \partial_\theta \tilde{b}(\theta; \theta) + K'(\theta), \\ (IR) & V(\theta) \geq \underline{U}, \quad \forall \theta, \\ (LL) & 0 \leq T(\theta), 0 \leq b(\theta) \leq \bar{b}, \quad \forall \theta. \end{cases}$$

*Technical detail.* See Lemma H.3 in Appendix H.

## 4.7 Optimal Transfer Schedule

*Technical detail.* See Lemma H.4 in Appendix H.

**Lemma 4.11** (Conditional cap–min calculus). *Let  $F_\beta(\cdot \mid \theta)$  be the c.d.f. of  $\beta(\hat{G})$  conditional on type with continuous density. For any cap  $b \geq 0$ ,*

$$\frac{\partial}{\partial b} \mathbb{E}[\min\{\beta(\hat{G}), b\} \mid \theta] = \mathbb{P}_\theta[\beta(\hat{G}) \geq b], \quad \frac{\partial}{\partial b} \mathbb{E}[\min\{\beta(\hat{G}), b\}^2 \mid \theta] = 2b \mathbb{P}_\theta[\beta(\hat{G}) \geq b].$$

**Proposition 4.12** (Closed-form optimal mechanism with statutory cap). *Let  $f(\theta) > 0$  and IFR on  $[\underline{\theta}, \bar{\theta}]$ , and let the provincial cost be  $C(x) = \alpha x + \frac{\kappa}{2}x^2$  in the realized payout  $x$ . In the threshold- $\beta$  baseline where  $\lambda_T \equiv \omega_T$ , the pointwise minimizer under  $0 \leq b \leq \bar{b}$  is*

$$b^*(\theta) = \min\left\{\bar{b}, \max\left\{0, \frac{\gamma \omega_b}{\kappa \omega_T} \frac{f(\theta)}{\bar{F}(\theta)} - \frac{\alpha}{\kappa}\right\}\right\}, \quad (4.6)$$

$$T^*(\theta) = T^*(\theta^{\min}) - \frac{\omega_b}{\omega_T} \left[ \tilde{b}^*(\theta) - \tilde{b}^*(\theta^{\min}) \right], \quad (4.7)$$

where  $\tilde{b}^*(\theta) = \mathbb{E}[\min\{\beta(\hat{G}), b^*(\theta)\} \mid \theta]$ ,  $\theta^{\min} = \inf\{\theta : b^*(\theta) > 0\}$ , and

$$\theta^\dagger = \inf\{\theta : b^*(\theta) = \bar{b}\}. \quad (4.8)$$

**Proposition 4.13** (No-bailout knife-edge). *Under Proposition 4.12's conditions, a self-consistent no bailout regime ( $b^*(\theta) \equiv 0$  for all  $\theta$ ) obtains iff*

$$\boxed{\alpha \omega_T \geq \gamma \omega_b \sup_{\theta \in [\underline{\theta}, \bar{\theta}]} h(\theta)}, \quad h(\theta) = f(\theta)/\bar{F}(\theta).$$

Otherwise the optimal cap is strictly positive on a set of positive measure.

**IR normalization and LL implications.** Normalize  $V(\theta^{\min}) = \underline{U}$  and note  $\tilde{b}^*(\theta^{\min}) = 0$ , whence (4.7) gives  $T^*(\theta^{\min}) = 0$ . Because of the negative relation in (4.7), the limited-liability requirement  $T \geq 0$  implies that whenever  $\tilde{b}^*(\theta) > 0$  on some region, the optimal  $T^*(\theta)$  is driven to the boundary  $T = 0$  there, shifting screening to  $b(\cdot)$ . Only when  $\gamma$  is sufficiently small relative to  $(\alpha, \kappa)$  can interior regions with  $T > 0$  arise.

## 4.8 Comparative statics

Let  $h(\theta) = f(\theta)/\bar{F}(\theta)$  and  $\lambda_T = \omega_T$  in the baseline. The interior zero solves  $h(\theta^{\min}) = \alpha \lambda_T / (\gamma \omega_b)$ . By the implicit function theorem,

$$\frac{\partial \theta^{\min}}{\partial \alpha} = \frac{1}{h'(\theta^{\min})} \frac{\lambda_T}{\gamma \omega_b} > 0, \quad \frac{\partial \theta^{\min}}{\partial \omega_b} = \frac{1}{h'(\theta^{\min})} \left( -\frac{\alpha \lambda_T}{\gamma \omega_b^2} \right) < 0,$$

$$\frac{\partial \theta^{\min}}{\partial \lambda_T} = \frac{1}{h'(\theta^{\min})} \frac{\alpha}{\gamma \omega_b} > 0, \quad \frac{\partial \theta^{\min}}{\partial \gamma} = \frac{1}{h'(\theta^{\min})} \left( -\frac{\alpha \lambda_T}{\gamma^2 \omega_b} \right) < 0.$$

For the interior cap  $b_{\max} = (\gamma \omega_b / \lambda_T - \alpha) / \kappa$ ,

$$\frac{\partial b_{\max}}{\partial \kappa} = -\frac{1}{\kappa^2} \left( \frac{\gamma \omega_b}{\lambda_T} - \alpha \right) < 0, \quad \frac{\partial b_{\max}}{\partial \lambda_T} = -\frac{\gamma \omega_b}{\kappa \lambda_T^2} < 0, \quad \frac{\partial b_{\max}}{\partial \gamma} = \frac{\omega_b}{\kappa \lambda_T} > 0.$$

## 4.9 Welfare Property

**Proposition 4.14** (Second-best efficiency). *Under Assumptions 4.1–4.9, the mechanism in Proposition 4.12 maximizes the expected sum of provincial and municipal utilities among all IC–IR–LL allocations.*

## 5 Policy Implications

Even though the present paper is purely theoretical, its closed-form solution delivers lessons that speak directly to provincial practice in NL and other fiscally stretched jurisdictions.

### 5.1 Design principles

**P1 Codify a *triple-zone* rule.** Proposition 4.12 together with (4.8) implies a simple menu in the baseline: (i) no transfer when the reported type is below  $\theta^{\min}$ ; (ii) a flat-to-rising cap on  $[\theta^{\min}, \theta^\dagger]$  following (4.6); (iii) a flat cap once type exceeds  $\theta^\dagger$ . Under a discretionary  $t=2$  rule (Appendix C), the realized payout becomes *threshold-linear-cap* in  $\hat{G}$ .

**P2 A single inequality decides whether bailouts survive.** Under the no bailout candidate, Proposition 4.13 shows that bailouts disappear when  $\alpha\omega_T \geq \gamma\omega_b \cdot \sup_{\theta} h(\theta)$ .

**P3 Front-load under softness (discretion).** When the  $t=2$  payout rule has an interior linear segment (Appendix C), the effective grant weight falls below  $\omega_T$  proportionally to the slope and the probability of being on that linear branch (Lemma 4.7). Softer rescue (steeper or more likely linear branch) thus calls for more front-loaded  $T$ ; cheaper provincial financing ( $\gamma \downarrow$ ) pushes in the opposite direction.

**P4 Make the cap bite by increasing  $\kappa$ .** The optimal cap  $b_{\max} = (\gamma\omega_b/\lambda_T - \alpha)/\kappa$  is inversely proportional to  $\kappa$ .

### 5.2 Political-economy robustness

**E1 Credibility of caps.** Without a hard legal ceiling, expectations rise, the discretion-based linear branch becomes more likely, the effective  $\lambda_T$  falls, and effort weakens.

**E2 Information precision.** Better accounting (lower  $\sigma_\eta$ ) reduces the measure of states on which the cap is slack at high signals and reinforces the effectiveness of the cap.

**E3 Vertical externalities.** The knife-edge in Proposition 4.13 offers a quantitative stress-test benchmark.

### 5.3 Limited-liability regions

From (4.7),  $T^*(\theta)$  is (weakly) decreasing in  $\tilde{b}^*(\theta)$ . Hence on any type region where  $\tilde{b}^*(\theta) > 0$ , the grant LL constraint ( $T \geq 0$ ) typically binds, pushing  $T^*(\theta)$  to 0 and shifting screening to  $b(\cdot)$ . Consequently, interior  $T^* > 0$  arises only (i) on the no bailout region where  $\tilde{b}^* = 0$  (i.e.  $\theta < \theta^{\min}$ ), or (ii) on ironed segments of the virtual weight when ironing is required under IFR.

When  $\gamma$  falls,  $b^*$  rises pointwise on its interior segment and the region with  $T^* = 0$  expands, unless  $\theta^{\min}$  shifts left enough to increase the no bailout mass.

## 5.4 Limitations and extensions

- L1 Static benchmark.** A multi-period version with learning would allow reputation-based caps and dynamic debt paths.
- L2 Heterogeneous  $\omega_b$ .** Allowing  $\omega_b(\theta)$  to vary with political or administrative quality could sharpen incentives (Appendix E).
- L3 Federal–provincial layer.** Embedding one more tier would capture Ottawa’s equalization backstop and test whether the knife-edge extends upward in the federation.

## 5.5 Implementation checklist and robustness

### A. Minimal implementation checklist.

1. **Fix primitives:** pick a parametric family for  $F(\theta)$  with IFR (e.g. log-logistic/exponential tail) and specify  $(\alpha, \kappa, \gamma, \omega_T, \omega_b, \bar{b})$  from budget documents or ranges.
2. **Compute hazard:**  $h(\theta) = f(\theta)/\bar{F}(\theta)$  and locate  $\theta^{\min}$  from  $h(\theta^{\min}) = \alpha \omega_T / (\gamma \omega_b)$ ; set  $\theta^\dagger$  by  $b^*(\theta) = \bar{b}$  as in (4.8).
3. **Construct the menu:**  $b^*(\theta)$  by (4.6) and  $T^*(\theta)$  by (4.7), then codify the triple-zone rule (no transfer / rising cap / flat cap).
4. **Audit rule:** if  $t=2$  discretion applies, use Appendix C to parameterize the linear slope and adjust the effective grant weight  $\lambda_T$  accordingly (cf. Lemma 4.7).

**B. Robustness flags.** The design principles rely on three modeling choices: (i) IFR for  $F$  (needed only for monotonicity/ironing); (ii) a nondecreasing, threshold-like  $\beta$  (administratively common); (iii) continuous signal noise (so boundary terms vanish and  $\lambda_T \equiv \omega_T$  under threshold  $\beta$ ). Under a discretionary linear branch,  $\lambda_T$  is scaled down by the branch slope times its probability; see Lemma 4.7 and Appendix C.

## 6 Conclusion

This paper recasts provincial–municipal rescues as a two-instrument screening problem with an implementable payout convention

$$p(\hat{G}, \hat{\theta}) = \mathbf{1}\{\hat{G} > 0\} \min\{\beta(\hat{G}), b(\hat{\theta}), \hat{G}\}.$$

Four takeaways emerge:

1. **One-dimensional reduction.** Under convexity and MLRP, the three-stage environment folds into a single-index screen in  $(T, b)$  (Proposition 4.8).
2. **Closed-form cap rule.** With linear-quadratic costs and a statutory cap, the IC-IR-LL optimum is *threshold-cap*, with cutoffs  $(\theta^{\min}, \theta^{\dagger})$  pinned down by the hazard  $h(\theta)$  (Proposition 4.12).
3. **Unified regime test.** A self-consistent no bailout regime obtains iff  $\alpha\omega_T \geq \gamma\omega_b \cdot \sup_{\theta} h(\theta)$ , providing a single marginal-cost benchmark (Proposition 4.13).
4. **Discretion vs. commitment.** Without commitment at  $t=2$ , the realized rule becomes *threshold-linear-cap*; the interior slope lowers the effective grant weight and strengthens soft budget incentives (Appendix C and Lemma 4.7).

*Limitations and next steps.* (i) Our baseline adopts IFR and threshold-like  $\beta$ ; when  $h(\theta)$  is nonmonotone, standard ironing applies and preserves implementability. (ii) A stylized calibration—even with public ranges for  $(\alpha, \kappa, \gamma, \omega_T, \omega_b, \bar{b})$ —would illustrate the triple-zone geometry and aid policy communication. (iii) Extending to a two-dimensional type or audit manipulation cost is feasible and would test the scope of the hazard-based cutoffs.

## 7 Acknowledgments

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# Appendices

## A Intergovernmental transfer instruments and local debt issuance

Table 1: Intergovernmental transfer instruments and local debt issuance.

Symbol	Economic content	Timing	Incentive effect
$\tau$	Unconditional grant (formula-based lump sum).	$t = 0$	Weak: arrives regardless of effort $e$ .
$g$	Unconditional block grant (fixed lump sum).	$t = 0$	Weak: independent of effort $e$ .
$sI$	Matching grant: province pays share $s$ of capital outlay $I$ .	$t = 0$ ( $s$ ), $t = 1$ ( $I$ )	Medium: requires local co-finance $(1 - s)I$ .
$p(\hat{G}, \hat{\theta})$	Ex post payout: $\mathbf{1}\{\hat{G} > 0\} \min\{\beta(\hat{G}), b(\hat{\theta}), \hat{G}\}$ .	$t = 2$	Strong: softens budget, lowers $e$ , raises $D$ .
$D$	Local debt issuance.	$t = 1$	Moderate: increases future $rD$ .



## B Symbols used in the model and empirical discussion

Table 2: Symbols used in the model and empirical discussion.

Symbol	Description
$i$	Local jurisdiction index.
$\theta$	Fiscal need / gap type (higher = weaker tax base, larger need).
$f(\theta), F(\theta)$	Density and c.d.f. of types on $[\underline{\theta}, \bar{\theta}]$ .
$\bar{F}(\theta)$	Survivor function $1 - F(\theta)$ .
$e$	Local revenue effort.
$R(e, \theta)$	Own-source revenue function.
$q$	Level of basic services.
$C(q, \theta)$	Cost to produce $q$ given $\theta$ .
$\tau$	Unconditional operating grant (MOG); maps to $T$ in the model.
$g$	Predictable capital transfer (CCBF).
$s$	Provincial cost-share rate for capital $I$ .
$I$	Local capital investment.
$D$	New debt (subject to approval); $\bar{D}$ debt limit.
$r$	Debt-service factor on $D$ .
$G$	Ex post fiscal gap before payout.
$\beta(\hat{G})$	Signal-based payout rule at $t=2$ .
$b(\theta)$	Type-based cap at $t=0$ .
$p(\hat{G}, \hat{\theta})$	Realized payout $\mathbf{1}\{\hat{G} > 0\} \min\{\beta(\hat{G}), b(\hat{\theta}), \hat{G}\}$ .
$T(\theta)$	Ex-ante grant schedule.
$\tilde{b}(\hat{\theta}; \theta)$	Expected payout under cap: $\mathbb{E}[\min\{\beta(\hat{G}), b(\hat{\theta})\} \mid \theta]$ .
$\omega_T, \omega_b$	Marginal utilities of $T$ and realized payout.
$\alpha, \kappa$	Payout cost parameters: $C(x) = \alpha x + \frac{\kappa}{2} x^2$ .
$\lambda_T(\theta)$	Effective weight on $T$ in screening: $\omega_T$ (threshold baseline).
$U_L$	Local government utility (interim).
$U_P$	Province's (negative) expected cost.
$V(\theta)$	Truthful utility $U_L(\theta, \theta)$ .
$\underline{U}$	Reservation utility (IR constraint).
$\theta^{\min}, \theta^{\dagger}$	Lower/upper cutoffs for the optimal cap.
$b_{\max}$	Interior bailout cap level.
$\rho$	Discount factor in the repeated game.
$\rho^*$	Critical discount factor.
$\pi$	Ex-ante default-coverage probability (descriptive).
$\delta$	Ex-ante default probability.
$\phi$	Welfare loss to residents under unresolved gap.
$\bar{b}$	Statutory cap on per-period bailout.
$\eta$	Audit/report noise.
$B(q), \Gamma(I)$	Utility/benefit from service level $q$ and investment $I$ .
$\chi$	Convex loss parameter in discretionary rescue (Appendix C).

## C Discretionary rescue at $t = 2$ and backward induction

**Provincial problem at  $t = 2$  (no commitment).** Suppose the Province cannot commit to  $\beta$  at  $t = 0$  and instead chooses a payout  $x$  at  $t = 2$  after observing the noisy gap signal  $\hat{G}$ . For tractability, let the loss from an unresolved residual gap  $(\hat{G} - x)_+$  be convex:

$$L((\hat{G} - x)_+) = \frac{\chi}{2} (\hat{G} - x)_+^2, \quad \chi > 0.$$

The Province solves

$$\min_{0 \leq x \leq \min\{\bar{b}, \hat{G}\}} \alpha x + \frac{\kappa}{2} x^2 + L((\hat{G} - x)_+).$$

When  $0 < x < \min\{\bar{b}, \hat{G}\}$  the FOC is  $\alpha + \kappa x - \chi(\hat{G} - x) = 0$ , hence

$$\beta^{\text{disc}}(\hat{G}) = \left[ \frac{\chi \hat{G} - \alpha}{\kappa + \chi} \right]_{[0, \bar{b}]}. \quad .$$

Therefore  $\beta^{\text{disc}}$  is *threshold-linear-cap* in  $\hat{G}$ .

**Backward induction to  $t = 1$ .** Municipalities at  $t = 1$  anticipate  $\beta^{\text{disc}}(\cdot)$  and choose effort accordingly. On the interior linear branch where  $\beta^{\text{disc}}(\hat{G}) = \chi/(\kappa + \chi)$ , the default-probability component in (4.3) is scaled by  $\kappa/(\kappa + \chi)$  and there is an additional marginal-rescue term  $-\omega_b \mathbb{E}[\beta^{\text{disc}}(\hat{G}) \mathbf{1}\{\beta^{\text{disc}} < b\}]$ .

**Discussion.** This discretionary benchmark microfound a threshold-linear-cap rule at  $t = 2$  and shows how the slope filters into (4.3), strengthening the soft budget moral-hazard channel. Our commitment baseline avoids time inconsistency by fixing  $\beta$  at  $t = 0$ ; the discretion variant is useful as a robustness check.

## D Dynamic credibility of no bailout

We now provide a complete proof of the credibility threshold

$$\rho^* = \frac{\omega_T - \omega_b}{\kappa + \gamma + \omega_T - \omega_b}$$

under a standard grim-trigger strategy profile in the infinitely repeated interaction between the Province and municipalities.

**Proposition D.1** (Dynamic credibility threshold). *Consider the infinite repetition ( $t = 0, 1, 2, \dots$ ) of the stage game whose commitment optimum under the*

no bailout candidate is  $b^*(\theta) \equiv 0$  for all  $\theta$ , with the induced  $T^*$  from Proposition 4.12. Suppose the Province discounts by  $\rho \in (0, 1)$  and adopts the following grim-trigger strategy: stay forever on the hard-budget path ( $p \equiv 0$ ) unless a bailout occurs in some period, in which case revert forever after to a soft regime in which municipalities anticipate bailouts and an allocatively inferior transfer mix must be used.<sup>5</sup>

Then the hard-budget path is a subgame perfect equilibrium (SPE) if and only if

$$\rho \geq \rho^* = \frac{\omega_T - \omega_b}{\kappa + \gamma + \omega_T - \omega_b}.$$

*Proof.* We compare the Province's one-shot deviation gain with the discounted loss from the continuation punishment.

*Normalization and notation.* Let  $x(\theta) = \lambda_T T(\theta) + \omega_b \tilde{b}(\theta; \theta)$  be the allocation index (Section 4.6). On the hard-budget path with threshold  $\beta$  we have  $\lambda_T \equiv \omega_T$  (Lemma 4.7), so a marginal increase of  $T$  by  $dT$  raises the local index by  $\omega_T dT$ , whereas (were a bailout allowed) a marginal payout  $dp$  would raise it by  $\omega_b dp$ .

*Step 1 (one-shot deviation gain bound).* Consider a deviation in some period  $\tau$  at an on-path history, where the Province pays a small realized bailout  $dp > 0$  to a set of marginal types with total measure normalized to one (so all “per-unit” statements below are in units of the allocation index). Because  $dp$  occurs at  $t=2$  within the stage and  $T$  is already fixed ex ante for that period, the only contemporaneous improvement available to the Province is the *instrument-substitution wedge*: relative to front-loading utility via  $T$ , delivering the same  $dx$  via  $p$  cannot save more than  $(\omega_T - \omega_b) dp$  units in the Lagrangian at the margin.<sup>6</sup> Hence, per unit of the marginal index delivered, the one-shot deviation gain is bounded by

$$\Delta_{\text{dev}} \leq \omega_T - \omega_b.$$

*Step 2 (punishment loss lower bound).* If a bailout occurs, grim-trigger prescribes reversion to a soft regime from the next period onward. In any such continuation, the Province must rely on a realized payout  $p > 0$  with positive probability on an interior set, incurring the convex resource cost  $\frac{\kappa}{2} p^2$  in addition to linear resource costs. Relative to the hard-budget path (where  $p \equiv 0$ ), the *per-period* increase in the Province's objective is at least the sum of (i) the marginal resource cost of front-loaded transfers  $\gamma$  that cannot be fully substituted away without violating IC at the hard-budget optimum, plus (ii) the convex incremental cost from using the  $p$ -instrument. Normalising by the same marginal unit of the allocation index as in Step 1 and using  $\kappa > 0$ , we obtain the conservative bound

$$L_{\text{pun}} \geq \kappa + \gamma.$$

<sup>5</sup>Formally, the reversion path can be any continuation equilibrium in which the realized payout is strictly positive on an interior set of histories/types with positive probability.

<sup>6</sup>Intuition: At the on-path hard-budget optimum, the IC/IR constraints are tight for some marginal types. Keeping those constraints satisfied, the most a deviator can gain in the current period by using  $p$  instead of  $T$  is bounded by the gap between their weights in the allocation index, because  $\lambda_T \equiv \omega_T$  and the  $p$ -channel enters the index with weight  $\omega_b$ . This is the same “instrument wedge” logic as in standard dynamic enforcement proofs with two instruments.

(We deliberately omit the nonnegative linear payout term  $\alpha p$  here; including it only raises  $L_{\text{pun}}$  and strengthens credibility.)

*Step 3 (incentive inequality and threshold).* The usual no-deviation condition under grim-trigger is

$$\Delta_{\text{dev}} \leq \frac{\rho}{1-\rho} L_{\text{pun}}.$$

Using the bounds from Steps 1–2 yields the *tight* inequality

$$\omega_T - \omega_b \leq \frac{\rho}{1-\rho} (\kappa + \gamma),$$

which is equivalent to

$$\rho \geq \frac{\omega_T - \omega_b}{\kappa + \gamma + \omega_T - \omega_b} = \rho^*.$$

This establishes that the hard-budget path is an SPE if and only if  $\rho \geq \rho^*$ .  $\square$

*Remark* (Conservativeness and robustness). (i) If we include the linear payout term  $\alpha p$  in the per-period punishment loss, then  $L_{\text{pun}} \geq \alpha + \kappa + \gamma$ , which *lowers*  $\rho^*$  and thus makes the hard-budget policy *easier* to sustain. Hence the threshold in Proposition D.1 is conservative. (ii) The argument requires only that the continuation after a deviation places positive probability on interior payouts (so that the convex part is operative); any such soft-regime continuation suffices.

## E Variable marginal utility of bailouts

When the marginal utility of a realized bailout,  $\omega_b(\theta)$ , varies across jurisdictions—for instance because political pressure is stronger for small communities—the first-order condition for the optimal cap becomes

$$\omega_b(\theta) = \alpha + \kappa b^*(\theta),$$

so that the linear segment in (4.6) reads

$$b^*(\theta) = \frac{\omega_b(\theta) - \alpha}{\kappa}, \quad 0 \leq b^*(\theta) \leq \bar{b}.$$

*Implication.* As long as  $\omega_b(\theta)$  is weakly increasing in  $\theta$ , the cap schedule remains monotone and retains the *threshold-linear-cap* geometry under the discretionary benchmark. The slope may now vary with type; for empirical calibration one needs an estimate of  $\omega_b(\theta)$ , e.g. from survey weights or past voting patterns.

## F Single-crossing and monotonicity details

With  $U_L(\hat{\theta}, \theta) = x(\hat{\theta}; \theta) + K(\theta)$  and  $\partial^2 U_L / \partial \theta \partial x \geq 0$ , the Spence–Mirrlees single-crossing property implies standard IC inequalities: for any  $\theta > \hat{\theta}$ ,

$$[U_L(\theta, \theta) - U_L(\hat{\theta}, \theta)] \geq [U_L(\theta, \hat{\theta}) - U_L(\hat{\theta}, \hat{\theta})].$$

Since  $K$  cancels, this reduces to  $x(\theta; \theta) - x(\hat{\theta}; \theta) \geq x(\theta; \hat{\theta}) - x(\hat{\theta}; \hat{\theta})$ . By letting reports be truthful on the RHS, we get  $x(\theta) \geq x(\hat{\theta})$ , hence monotonicity. When the virtual term  $(\gamma\omega_b/\lambda_T(\theta))h(\theta)$  fails to be increasing, standard ironing (à la Myerson) delivers a nondecreasing ironed index.

## G Regularity for differentiation under the expectation

We justify the steps leading to (4.3) and Lemma H.2.

**Dominated convergence / Leibniz rule.** Assume: (i)  $\eta$  has a continuous density  $f_\eta$  with bounded tails; (ii)  $\beta$  is piecewise  $C^1$  with slope in  $[0, 1]$  and bounded image; (iii)  $R'_e(e, \theta)$  is continuous and locally bounded uniformly in  $e$  on compact sets. Then, for any integrable function  $g(\hat{G}, e)$  that is piecewise  $C^1$  in  $e$  and dominated by an integrable envelope, we may differentiate inside the expectation by dominated convergence / Leibniz's rule:

$$\frac{\partial}{\partial e} \mathbb{E}[g(\hat{G}, e)] = \mathbb{E}\left[\frac{\partial}{\partial e} g(\hat{G}, e)\right].$$

**Indicators and boundary sets.** For events of the form  $\{\hat{G} - \beta(\hat{G}) > 0\}$ , the boundary  $\{\hat{G} - \beta(\hat{G}) = 0\}$  has Lebesgue measure zero because  $\eta$  has a density and  $\beta$  is a.e. differentiable with bounded slope; hence the derivative of the indicator contributes no boundary term. On threshold rules,  $\beta'(\hat{G}) = 0$  a.e., so the marginal-rescue term vanishes, yielding the expressions stated in Lemma H.2 and (4.3).

## H Technical Lemmas and Proofs

*Note (implementable payout).* Throughout, the realized payout is  $p(\hat{G}, \hat{\theta}) = \mathbf{1}\{\hat{G} > 0\} \min\{\beta(\hat{G}), b(\hat{\theta}), \hat{G}\}$ . On the cap-slack and positive-signal set where  $\beta(\hat{G}) < b(\hat{\theta})$ , all derivatives below coincide with those under  $p = \beta(\hat{G})$ . When the min picks  $\hat{G}$ , boundary sets have Lebesgue measure zero under continuous noise, so the derivative contributions vanish a.e.

**Lemma H.1** (Effort independence from report). *Under Assumptions 4.1, 4.2 and 4.4, the interior first-order condition (4.3) can be rewritten*

$$R'_e(e^*(\theta), \theta) \{1 + \varphi \Lambda\} = \phi, \quad \Lambda = \mathbb{E}[f_\eta(\hat{G} - \beta(\hat{G}))].$$

*All terms on the right depend only on the true type  $\theta$ ; hence  $e^*(\theta)$  is independent of the reported  $\hat{\theta}$  up to boundary-density terms on the cap-binding tail.*

**Lemma H.2** (Marginal default probability on the signal branch). *Under Assumptions 4.1–4.2, let  $\delta = \mathbb{P}_\theta[p(\hat{G}, \hat{\theta}) < G]$ . On the set where  $\beta(\hat{G}) < b(\hat{\theta})$  (cap slack),*

$$\frac{\partial \delta}{\partial e} = -R'_e(e, \theta) \mathbb{E} \left[ f_\eta(\hat{G} - \beta(\hat{G})) (1 - \beta'(\hat{G})) \right],$$

*and for threshold rules (piecewise constant  $\beta$ ),  $\beta'(\hat{G}) = 0$  a.e., so  $\frac{\partial \delta}{\partial e} = -R'_e(e, \theta) \mathbb{E} [f_\eta(\hat{G} - \beta(\hat{G}))]$ .*

*Proof of Lemma 4.7.* Recall  $\hat{G} = G + \eta$  and  $\partial_T \hat{G} = \partial_T G = -1$ . Write

$$p(\hat{G}, \hat{\theta}) = \mathbf{1}\{\hat{G} > 0\} \min\{\beta(\hat{G}), b(\hat{\theta}), \hat{G}\}.$$

Since  $\beta(0) = 0$  and  $\beta' \in [0, 1]$ , for  $\hat{G} \geq 0$  we have  $\beta(\hat{G}) \leq \hat{G}$ . Hence on  $\{\hat{G} > 0\}$  and cap-slack  $\{\beta(\hat{G}) < b(\hat{\theta})\}$ , the minimum is  $\beta(\hat{G})$  and

$$\partial_T p = \beta'(\hat{G}) \partial_T \hat{G} = -\beta'(\hat{G}).$$

On the cap-binding set  $\{\beta(\hat{G}) \geq b(\hat{\theta})\}$ ,  $p = b(\hat{\theta})$  so  $\partial_T p = 0$ . Therefore,

$$\frac{\partial}{\partial T} \mathbb{E}[p(\hat{G}, \hat{\theta}) \mid \theta] = -\mathbb{E} \left[ \beta'(\hat{G}) \mathbf{1}\{\beta(\hat{G}) < b(\hat{\theta})\} \mid \theta \right] + \text{boundary terms}.$$

The boundary terms arise only when  $\operatorname{argmin}\{\beta(\hat{G}), b(\hat{\theta}), \hat{G}\}$  changes; since  $\eta$  has a continuous density and  $\beta$  is a.e. differentiable with slope  $< 1$ , those switch sets have Lebesgue measure zero and their contribution vanishes under dominated convergence. Hence  $\lambda_T(\theta) = \omega_T - \omega_b \partial_T \mathbb{E}[p \mid \theta]$  reduces to the stated expressions; in particular, for threshold  $\beta$  we obtain  $\lambda_T(\theta) \equiv \omega_T$ .  $\square$

**Lemma H.3** (Monotonicity of the allocation index). *Under IC and Assumption 4.9, the implemented allocation index*

$$x(\theta) = \lambda_T(\theta) T(\theta) + \omega_b(\theta) \tilde{b}(\theta; \theta)$$

*is weakly increasing in  $\theta$ . When ironing is required under IFR, the ironed allocation is nondecreasing.*

**Lemma H.4** (Monotonicity under IFR and caps). *If  $\lambda_T$  and  $\omega_b$  are locally constant and  $f/\bar{F}$  is increasing (IFR), then the optimal cap  $b^*(\theta) = \min\{\bar{b}, \max\{0, \tilde{b}(\theta)\}\}$  is weakly increasing, where  $\tilde{b}(\theta) = \frac{\gamma \omega_b}{\kappa \lambda_T} \frac{f(\theta)}{\bar{F}(\theta)} - \frac{\alpha}{\kappa}$ . If  $\lambda_T(\theta)$  varies, a sufficient condition is that  $\lambda_T(\theta)$  is weakly decreasing and  $f(\theta)/\bar{F}(\theta)$  is increasing; otherwise, apply standard ironing on the virtual term  $\frac{\gamma \omega_b}{\lambda_T(\theta)} \frac{f(\theta)}{\bar{F}(\theta)}$ .*

*Proof of Eq. (4.3) (first-order condition for  $e$ ).* Fix  $(\tau, s, \bar{D}, \beta, b)$  and true type  $\theta$ . The municipality's  $t=1$  objective as a function of  $e$  (dropping terms independent of  $e$ ) is

$$\Phi(e) = \mathbb{E} \left[ R(e, \theta) - \phi e - \varphi \mathbf{1}\{p(\hat{G}, \hat{\theta}) < G\} + \omega_b p(\hat{G}, \hat{\theta}) \right],$$

with  $\hat{G} = G(e) + \eta$  and  $G(e) = C(\cdot) - [R(e, \theta) + \tau + g] + \dots$  so that  $\partial_e \hat{G} = \partial_e G = -R'_e(e, \theta)$ .

**Step 1 (Justifying differentiation under  $\mathbb{E}$ ).** By Assumptions 4.2–4.4,  $\eta$  has a continuous density  $f_\eta$  with bounded tails,  $\beta$  is piecewise  $C^1$  with slope in  $[0, 1)$  and bounded image, and  $R'_e$  is continuous and locally bounded. Hence all integrands below admit a uniform integrable envelope, so dominated convergence / Leibniz rule applies and we may interchange  $\partial_e$  and  $\mathbb{E}$ .

**Step 2 (Derivative of the default indicator).** Define  $H(e, \eta) = \hat{G} - \beta(\hat{G})$ . On the set where  $\beta$  is differentiable,

$$\partial_e H(e, \eta) = (\partial_e \hat{G}) (1 - \beta'(\hat{G})) = -R'_e(e, \theta) (1 - \beta'(\hat{G})).$$

Approximate the Heaviside  $\mathbf{1}\{u > 0\}$  by smooth  $s_n(u)$  with  $s'_n \rightarrow \delta_0$  in the sense of distributions, and apply dominated convergence:

$$\frac{\partial}{\partial e} \mathbb{E}[\mathbf{1}\{H(e, \eta) > 0\}] = \lim_{n \rightarrow \infty} \mathbb{E}[s'_n(H) \partial_e H] = \mathbb{E}[\delta_0(H) \partial_e H].$$

Since  $H = \hat{G} - \beta(\hat{G})$  is a monotone  $C^1$  transformation of  $\hat{G}$  with slope  $1 - \beta'(\hat{G}) \in (0, 1]$  a.e., the density of  $H$  at 0 equals  $f_\eta(\hat{G} - \beta(\hat{G}))$  a.e. Hence

$$\frac{\partial}{\partial e} \mathbb{E}[\mathbf{1}\{p(\hat{G}, \hat{\theta}) < G\}] = \frac{\partial}{\partial e} \mathbb{E}[\mathbf{1}\{H > 0\}] = -R'_e(e, \theta) \mathbb{E}[f_\eta(\hat{G} - \beta(\hat{G})) (1 - \beta'(\hat{G}))],$$

where we have used that on the cap-slack set  $\{\beta(\hat{G}) < b(\hat{\theta})\}$  the event  $\{p < G\}$  coincides with  $\{H > 0\}$ , while on the cap-binding set the boundary-density contribution is  $O(\varepsilon)$  by Assumption 4.6 and does not affect the sign/comparative statics.

**Step 3 (Derivative of the realized payout).** Write  $p(\hat{G}, \hat{\theta}) = \mathbf{1}\{\hat{G} > 0\} \min\{\beta(\hat{G}), b(\hat{\theta}), \hat{G}\}$ . Since  $\beta(0) = 0$  and  $\beta' \in [0, 1)$ , we have  $\beta(\hat{G}) \leq \hat{G}$  for all  $\hat{G} \geq 0$ ; thus on  $\{\hat{G} > 0\}$  and cap-slack  $\{\beta(\hat{G}) < b(\hat{\theta})\}$ ,  $p = \beta(\hat{G})$  and

$$\partial_e p = \beta'(\hat{G}) \partial_e \hat{G} = -\beta'(\hat{G}) R'_e(e, \theta).$$

On the cap-binding set  $p = b(\hat{\theta})$  so  $\partial_e p = 0$ ; hence

$$\frac{\partial}{\partial e} \mathbb{E}[p(\hat{G}, \hat{\theta})] = -R'_e(e, \theta) \mathbb{E}[\beta'(\hat{G}) \mathbf{1}\{\beta(\hat{G}) < b(\hat{\theta})\}] + O(\varepsilon).$$

**Step 4 (FOC).** Collecting terms,

$$\Phi'(e) = R'_e(e, \theta) - \phi - \varphi \frac{\partial}{\partial e} \mathbb{E}[\mathbf{1}\{p < G\}] + \omega_b \frac{\partial}{\partial e} \mathbb{E}[p].$$

Using the expressions above and canceling the common factor  $R'_e(e, \theta)$ , the  $O(\varepsilon)$  terms vanish by Assumption 4.6, and the first-order condition  $\Phi'(e^*) = 0$  becomes

$$R'_e(e^*(\theta), \theta) \left\{ 1 + \varphi \mathbb{E}[f_\eta(\hat{G} - \beta(\hat{G})) (1 - \beta'(\hat{G}))] - \omega_b \mathbb{E}[\beta'(\hat{G}) \mathbf{1}\{\beta(\hat{G}) < b(\hat{\theta})\}] \right\} = \phi,$$

which is Eq. (4.3).  $\square$

*Proof of Lemma H.2.* Let  $\delta(e) = \mathbb{P}_\theta[p(\hat{G}, \hat{\theta}) < G]$ . On the cap-slack event  $\{\beta(\hat{G}) < b(\hat{\theta})\}$  we have  $\{p < G\} = \{\hat{G} - \beta(\hat{G}) > 0\} = \{H > 0\}$ . Repeating the mollifier argument in the proof of Eq. (4.3),

$$\delta'(e) = \frac{\partial}{\partial e} \mathbb{E}[\mathbf{1}\{H > 0\}] = \mathbb{E}[\delta_0(H) \partial_e H] = -R'_e(e, \theta) \mathbb{E}\left[f_\eta(\hat{G} - \beta(\hat{G})) (1 - \beta'(\hat{G}))\right].$$

For threshold  $\beta$  we have  $\beta'(\hat{G}) = 0$  a.e., hence

$$\delta'(e) = -R'_e(e, \theta) \mathbb{E}\left[f_\eta(\hat{G} - \beta(\hat{G}))\right].$$

On the cap-binding set, the event  $\{p < G\}$  becomes  $\{b(\hat{\theta}) < G\}$  and contributes only a boundary-density term with probability  $O(\varepsilon)$ , which does not alter the formula.  $\square$

*Proof of Lemma 4.11.* Let  $Y = \beta(\hat{G})$  with c.d.f.  $F_\beta(\cdot | \theta)$  and continuous density. Then

$$\mathbb{E}[\min\{Y, b\} | \theta] = \int_0^\infty \min\{y, b\} dF_\beta(y | \theta) = \int_0^b y dF_\beta + \int_b^\infty b dF_\beta.$$

Differentiating w.r.t.  $b$  and using the fundamental theorem for Lebesgue integrals gives

$$\partial_b \mathbb{E}[\min\{Y, b\} | \theta] = \mathbb{P}_\theta[Y \geq b].$$

Similarly,

$$\mathbb{E}[\min\{Y, b\}^2 | \theta] = \int_0^b y^2 dF_\beta + \int_b^\infty b^2 dF_\beta = \int_0^b \mathbb{P}_\theta[Y \geq t] (2t) dt,$$

so  $\partial_b \mathbb{E}[\min\{Y, b\}^2 | \theta] = 2b \mathbb{P}_\theta[Y \geq b]$ .  $\square$

*Proof of Proposition 4.8 (reduction to quasi-linearity).* Fix  $(\tau, s, \bar{D}, \beta)$  and true type  $\theta$ . Minimizing over  $(q, I, D)$  yields reduced cost  $C_0(\theta)$  independent of the report. By Assumption 4.5, the interior  $e^*(\theta)$  is report-independent up to  $O(\varepsilon)$  boundary terms. Hence interim utility can be written as

$$U_L(\hat{\theta}, \theta) = \lambda_T(\theta) T(\hat{\theta}) + \omega_b \tilde{b}(\hat{\theta}; \theta) + K(\theta),$$

with  $\lambda_T(\theta) = \omega_T - \omega_b \partial_T \mathbb{E}[p(\hat{G}, \hat{\theta}) | \theta]$  and  $\tilde{b}(\hat{\theta}; \theta) = \mathbb{E}[\min\{\beta(\hat{G}), b(\hat{\theta})\} | \theta]$ . By Lemma 4.7, under threshold  $\beta$  and continuous noise the boundary terms vanish and  $\partial_T \mathbb{E}[p | \theta] = 0$ , hence  $\lambda_T(\theta) = \omega_T$  a.e. This delivers the quasi-linear reduced form (4.4).  $\square$

*Proof of Lemma H.3.* Let  $x(\hat{\theta}; \theta) = \lambda_T(\theta) T(\hat{\theta}) + \omega_b(\theta) \tilde{b}(\hat{\theta}; \theta)$  and  $U_L(\hat{\theta}, \theta) = x(\hat{\theta}; \theta) + K(\theta)$ . By Assumption 4.9,  $\partial^2 U_L / \partial \theta \partial x \geq 0$  (Spence–Mirrlees). Suppose, towards a contradiction, that there exist  $\theta_2 > \theta_1$  with  $x(\theta_2) < x(\theta_1)$ . Then IC implies

$$U_L(\theta_2, \theta_2) \geq U_L(\theta_1, \theta_2), \quad U_L(\theta_1, \theta_1) \geq U_L(\theta_2, \theta_1).$$



Subtracting and using single crossing yields  $x(\theta_2) \geq x(\theta_1)$ , a contradiction. Hence  $x(\theta)$  is weakly increasing. When ironing is needed (IFR with nonmonotone virtual term), the ironed allocation preserves nondecreasingness.  $\square$

*Proof of Lemma H.4.* Fix  $\theta$  and consider an incremental increase of  $b(\theta)$  by  $db$  while holding  $b$  elsewhere fixed. By Lemma 4.11, the marginal increase in the Province's expected cost at type  $\theta$  equals

$$(\alpha + \kappa b(\theta)) \mathbb{P}_\theta[\beta(\hat{G}) \geq b(\theta)] db.$$

By Myerson's envelope for direct mechanisms, the marginal (virtual) benefit from relaxing the cap at  $\theta$  equals

$$\frac{\gamma}{\lambda_T(\theta)} \omega_b(\theta) h(\theta) \mathbb{P}_\theta[\beta(\hat{G}) \geq b(\theta)] db,$$

where  $h(\theta) = f(\theta)/\bar{F}(\theta)$  under IFR. Equating marginal cost and benefit cancels the common tail probability and yields the pointwise KKT condition

$$\alpha + \kappa b(\theta) = \frac{\gamma \omega_b(\theta)}{\lambda_T(\theta)} h(\theta).$$

If  $\lambda_T, \omega_b$  are locally constant, this implies  $b(\theta) = \frac{1}{\kappa} \left( \frac{\gamma \omega_b}{\lambda_T} h(\theta) - \alpha \right)$ , which is increasing in  $\theta$  because  $h$  is increasing under IFR. Projection onto  $[0, \bar{b}]$  preserves weak monotonicity. If  $\lambda_T(\theta)$  varies with  $\theta$ , a sufficient condition for the RHS to be weakly increasing is that  $\lambda_T$  be weakly decreasing while  $h$  is weakly increasing; otherwise standard ironing of the virtual term  $\frac{\gamma \omega_b}{\lambda_T(\theta)} h(\theta)$  restores a nondecreasing  $b^*(\cdot)$ .  $\square$

*Proof of Proposition 4.12.* Work with the reduced form, threshold  $\beta$ , and IFR so that ironing yields a nondecreasing allocation. The Province minimizes

$$\mathbb{E}_\theta \left[ \mathbb{E} \left[ \alpha \min\{\beta(\hat{G}), b(\theta)\} + \frac{\kappa}{2} \min\{\beta(\hat{G}), b(\theta)\}^2 \mid \theta \right] + \gamma T(\theta) \right]$$

subject to IC/IR/LL and monotonicity. Using Lemma 4.11, the pointwise marginal cost (at type  $\theta$ ) of increasing  $b(\theta)$  equals  $(\alpha + \kappa b(\theta)) \mathbb{P}_\theta[\beta \geq b(\theta)]$ . By Myerson's lemma, the virtual marginal benefit equals  $(\gamma/\lambda_T) \omega_b h(\theta) \mathbb{P}_\theta[\beta \geq b(\theta)]$  with  $h(\theta) = f/\bar{F}$ . Equating and canceling the common tail probability gives the interior solution

$$b(\theta) = \frac{1}{\kappa} \left( \frac{\gamma \omega_b}{\lambda_T} h(\theta) - \alpha \right).$$

Projection onto  $[0, \bar{b}]$  yields (4.6), and IFR implies monotonicity (Lemma H.4). To recover  $T^*$ , note that under  $\lambda_T \equiv \omega_T$  the implemented index  $x(\theta) = \omega_T T(\theta) + \omega_b \tilde{b}(\theta; \theta)$  must be nondecreasing; holding  $x$  feasible implies  $dT = -(\omega_b/\omega_T) d\tilde{b}$  a.e., so integrating from  $\theta^{\min}$  (where  $\tilde{b}^* = 0$  by definition) gives (4.7) with  $T^*(\theta^{\min}) = 0$  by IR normalization.  $\square$

*Proof of Proposition 4.13.* At  $b \equiv 0$ , the marginal expected cost of relaxing the cap at  $\theta$  is  $\alpha$  (Lemma 4.11), while the virtual marginal benefit equals  $(\gamma/\lambda_T)\omega_b h(\theta) = (\gamma\omega_b/\omega_T)h(\theta)$  under the threshold- $\beta$  baseline. If  $\alpha \geq (\gamma\omega_b/\omega_T)h(\theta)$  for all  $\theta$ , then the KKT condition is nonnegative everywhere and  $b = 0$  is pointwise optimal. Otherwise at any  $\theta^\# \in \arg \max h(\theta)$  with strict inequality, increasing  $b(\theta^\#)$  strictly reduces the objective, so  $b^* \equiv 0$  cannot be optimal.  $\square$

*Proof of Proposition 4.14.* Total expected welfare equals the sum of municipal interim utilities minus provincial costs:

$$W(T, b) = \mathbb{E}_\theta[V(\theta)] - \mathbb{E}_\theta\left[\gamma T(\theta) + \mathbb{E}\{\alpha p + \frac{\kappa}{2}p^2 \mid \theta\}\right], \quad p = \min\{\beta(\hat{G}), b(\theta)\}.$$

Under IC with  $V(\underline{\theta}) = \underline{U}$ , the envelope formula (Remark 4.6) gives

$$V'(\theta) = \lambda'_T(\theta)T(\theta) + \omega_b \partial_\theta \tilde{b}(\theta; \theta) + K'(\theta).$$

Integrating and substituting into  $W$  yields a virtual-surplus functional in which the  $\theta$ -wise marginal effect of  $b(\theta)$  is exactly the difference between the virtual benefit  $(\gamma/\lambda_T)\omega_b h(\theta)$  and the marginal expected cost  $\alpha + \kappa b(\theta)$  (times the common tail probability). Hence maximizing  $W$  subject to IC/IR/LL and monotonicity is equivalent to the pointwise KKT condition used in Proposition 4.12; the resulting allocation is therefore second-best efficient among all IC-IR-LL mechanisms.  $\square$

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